

# Can brane dark energy model be probed observationally by distant supernovae?

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## Abstract

The recent astronomical measurements of distant supernovae as well as other observations indicate that our universe is presently accelerating. There are different proposals for the explanation of this acceleration, such as the cosmological constant  $\Lambda$ , decaying vacuum energy, an evolving scalar field (quintessence), phantom energy, etc. Most of these proposals require the existence of exotic matter with negative pressure violating the strong energy condition. On the other hand, there have appeared many models which offer dramatically different mechanisms for the current acceleration, in which dark energy emerges from the gravity sector rather than from the matter sector. In this paper, we compare the concordance  $\Lambda$ CDM model with the Sahni-Shtanov brane-world models of dark energy by using the Akaike and Bayesian information criteria. We show that new parameters in the brane model are not statistically significant in terms of the information criteria, although the best fit method gives an improved fit to the SNIa data, because of the additional parameters. This is because the information criteria of model selection compensate for this advantage by penalizing models having more free parameters. We conclude that only new future observational data are accurate enough to give an advantage to dark-energy models of the brane origin, i.e., a very high-significance detection is required to justify the presence of new parameters. In our statistical analysis both Riess et al.'s and Astier et al.'s SNIa samples are used. For stringent constraining parameters of the models the baryon oscillation peak (BOP) test is used.

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## 1 Introduction

The recent supernovae SNIa measurements [3,4] as well as other observations indicate that the expansion of our present universe is accelerating. While the cosmological constant offers the possibility of effective explanation of the acceleration, the existence of fine tuning difficulties motivate theorist to search

for alternative forms of dark energy. All these proposals can be divided into two groups following the criterion whether dark energy emerges from gravity or matter sectors of the theory. The first group is characterized by postulating the existence of unusual properties of matter content with negative pressure violating the strong energy condition. This category of models includes (besides the  $\Lambda$ CDM model or varying  $\Lambda$ -term model) the quintessence models (referred to by some as models of an evolving scalar field), phantom models, etc. As a representative model of this class, we consider the  $\Lambda$ CDM model which we confront with a subclass of models of the second category—the brane models of dark energy. In these models, dark energy emerges from a different evolutionary scenario at the late time of evolution. In this approach, instead of a new hypothetical energy component of an unknown form, dark energy arises from the modified gravity sector of the theory. The basic idea in these cosmologies is that our observable Universe is a four-dimensional brane embedded in a five-dimensional bulk space. As the representatives of this cosmologies, we consider two classes of models which appeared in recent achievement, namely (i) the Deffayet-Dvali-Gabadadze (DDG) model [6,7,8] and (ii) the Sahni-Shtanov (SSh) model [1,2]. The main difference between these two models is that in the second one includes both brane and bulk cosmological constants and, similarly to the DDG model, also includes the scalar curvature term in the action for the brane. The Randall-Sundrum (RS) model [9,10] can be recovered as a special limit of the SSh model. Thus, the SSh model generalizes both the RS model and the DDG model. Compared to general relativity, both the DDG and SSh models introduce some extra parameters, and then it is crucial to perform an objective comparison of these models. In the generic case, introducing extra parameters result naturally in an improved fit to the data, but a crucial question is whether these new parameters are actually relevant for explaining SNIa data set [11].

One of the most popular procedures adopted to compare models with a different parameters is to use the best-fit method based on the maximum of the likelihood function. However, it is well known that this method favours the model with the largest number of parameters. The likelihood ratio test [12] based on the simplest procedure of calculating the quantity  $2 \ln \frac{\mathcal{L}_{\text{simple}}}{\mathcal{L}_{\text{complex}}}$  (where  $\mathcal{L}$  is the maximum likelihood) can also be useful in this context. While this ratio can be used to control the significance of any increase in the likelihood against an additional parameter introduced to the model, the assumptions of this criterion are often violated in the astrophysical applications [13].

The key aim of this letter is to make an objective comparison of two different groups of dark energy models which may feature different numbers of model parameters. We use the Akaike information criteria (AIC) [14] and the Bayesian information criteria (BIC) [15] to select model parameters providing the preferred fit to data. These information criteria enable us to select the combination of cosmological parameters giving the best fit to the present

SN Ia data. Of course, some future observational data (from SNAP, for example) may give arguments in favour of additional parameters of brane-world dark energy, but here we claim that such models have no impact on the current Universe. Taking into account the simplicity argument, we argue that for the verification of the idea of brane-world dark energy very high significance detection is required and, therefore, at present these extra parameters have a marginal significance in the fits to the present data.

## 2 Cosmologies with brane dark energy origin

It was pointed out by many authors [16,17,18] that brane models offer a wider range of possibilities for solving the problem of acceleration than standard  $\Lambda$ CDM model. Alam and Sahni [19] claimed that the “Brane (1)” model, which has the effective equation of state  $w \equiv p/\rho < -1$ , provides better agreement with the SN Ia data than the  $\Lambda$ CDM model for matter density parameter  $\Omega_{m,0} > 0.3$  (and for  $\Omega_{m,0} \leq 0.25$ ). Such a conclusion comes from a simple comparison of the best-fit method based on the maximum likelihood function which usually favour models with the largest number of parameters, in our case, the SSh model over the  $\Lambda$ CDM model. In Table 1, two different brane models and the reference  $\Lambda$ CDM model are represented in terms of the Hubble parameter  $H$  as a function of redshift  $z$ . For simplicity we considered the flat case  $\Omega_{k,0} = 0$  which is strongly preferred by the WMAP data [20]. Also in our previous paper [21,22] we find that, when we analyze fit to SN Ia data, in general cases the number of essential parameters in the cosmological models with dark energy is in principal, equal to two, namely,  $H_0$ ,  $\Omega_{m,0}$ , i.e.,  $\Omega_{k,0} = 0$  is not an essential parameter.

The presence of two bulk and brane cosmological constants distinguishes the SSh model from the DDG model (see Table 1). In the terminology of Sahni and Shtanov the SSh model is called the Brane (1) model according to different ways of bounding anti de Sitter (or Schwarzschild) bulk space by the brane [23]). The decaying  $C/a^4$  dark radiation term can be neglected in the basic Friedmann equation [24]. The generalized Friedmann equation assumes the following form

$$H^2 + \frac{k}{a^2} = \frac{\Lambda_b}{6} + \frac{C}{a^4} + \frac{1}{l^2} \left( \sqrt{1 + l^2 \left( \frac{\rho + \sigma}{3m^2} - \frac{\Lambda_b}{6} - \frac{C}{a^4} \right)} \mp 1 \right)^2 \quad (1)$$

where  $l = \frac{2m^2}{M^2}$  is the length scale,  $m^2$  is the coupling constant in action  $M$  and  $m$  denote the five and four dimensional Planck masses respectively,  $\Lambda_b$  is the cosmological constant on the bulk,  $\sigma$  is the brane tension, and the plus/minus sign before the last term corresponds to “Brane (1)” and “Brane (2)” solution, respectively.

In Table 1, equation (1) is expressed in terms of dimensionless density parameters for the flat case  $\Omega_{m,0} = \frac{\rho_0}{3m^2 H_0^2}$ ,  $\Omega_{\sigma,0} = \frac{\sigma}{3m^2 H_0^2}$ ,  $\Omega_{l,0} = \frac{1}{l^2 H_0^2}$ ,  $\Omega_{\Lambda_b,0} = -\frac{\Lambda_b}{6H_0^2}$ .

For completeness we consider the flat brane cosmology which bases on Defayet's modification of the FRW equation [8,25]

$$H^2 \pm \frac{H}{r_0} = \frac{\rho}{3} + \frac{\Lambda}{3} \quad (2)$$

where  $r_0 = \mathcal{M}_{Pl}^2/2M^3$  is the scale on which it is possible to “probe” the extra dimension, the plus/minus sign corresponds the two distinct cosmological phases—the self accelerating and Minkowski cosmological ones. This model, which fits well to SNIa data, was analysed by Lue and Starkman [25]. This model belongs to a class of models can be derived from the SSh models after setting the bulk cosmological constant to zero. Therefore these models are classified as SSh( $\Lambda_b = 0$ ).

In Table 1 we complete all these models together with the dependence of Hubble's function  $H = \frac{\dot{a}}{a}$  on redshift. We also denote the number of a model's independent, free parameters by  $d$ . Note that we have constraint on all rewritten  $\Omega_{i,0}$  parameters from the condition  $H(z = 0) = H_0$ .

### 3 The Akaike and Bayesian information criteria

The information criteria (in a similar way as the adjusted coefficient of determination in standard statistics) put a threshold which must be exceeded in order to assert an additional parameter to be important in explanation of the phenomenon. The discussion how high this threshold should be caused appearing many different criteria. The Akaike and Bayesian information criteria (AIC and BIC) (for review see [26]) are most popular and used in everyday statistical practices.

In the case of the model in question we find that the AIC and BIC information criteria of model selection do not provide sufficient arguments for incorporation of new parameters from brane cosmology when SNIa data are used. It is in contrast to the conclusion which Alam and Sahni [19,27]) obtained without using the information criteria.

The usefulness of using information criteria of model selection was recently demonstrated by Liddle [11] and Parkinson et al. [28]

The AIC is defined in the following way [14]

$$\text{AIC} = -2 \ln \mathcal{L} + 2d \quad (3)$$

case	model	$H(z)$	model parameters	$d$
1	$\Lambda$ CDM	$H = H_0 \sqrt{\Omega_{\text{m},0}(1+z)^3 + \Omega_{\Lambda,0}}$	$H_0, \Omega_{\text{m},0}, \Omega_{\Lambda,0}$	2
2	DDG	$H = H_0 \sqrt{(\sqrt{\Omega_{\text{m},0}(1+z)^3 + \Omega_{\text{rc},0}} + \sqrt{\Omega_{\text{rc},0}})^2}$	$H_0, \Omega_{\text{m},0}, \Omega_{\text{rc},0}$	2
3a	SSh Brane 1	$H = H_0 \sqrt{\Omega_{\text{m},0}(1+z)^3 + \Omega_{\sigma,0} + 2\Omega_{\text{l},0} - 2\sqrt{\Omega_{\text{l},0}}\sqrt{\Omega_{\text{m},0}(1+z)^3 + \Omega_{\sigma,0} + \Omega_{\text{l},0} + \Omega_{\Lambda_{\text{b}},0}}}$	$H_0, \Omega_{\text{m},0}, \Omega_{\sigma,0}, \Omega_{\text{l},0}, \Omega_{\Lambda_{\text{b}},0}$	4
3b	SSh Brane 2	$H = H_0 \sqrt{\Omega_{\text{m},0}(1+z)^3 + \Omega_{\sigma,0} + 2\Omega_{\text{l},0} + 2\sqrt{\Omega_{\text{l},0}}\sqrt{\Omega_{\text{m},0}(1+z)^3 + \Omega_{\sigma,0} + \Omega_{\text{l},0} + \Omega_{\Lambda_{\text{b}},0}}}$	$H_0, \Omega_{\text{m},0}, \Omega_{\sigma,0}, \Omega_{\text{l},0}, \Omega_{\Lambda_{\text{b}},0}$	4
4a	SSh1( $\Lambda_b = 0$ )	$H = H_0 \sqrt{\Omega_{\text{m},0}(1+z)^3 + \Omega_{\sigma,0} + 2\Omega_{\text{l},0} - 2\sqrt{\Omega_{\text{l},0}}\sqrt{\Omega_{\text{m},0}(1+z)^3 + \Omega_{\sigma,0} + \Omega_{\text{l},0}}}$	$H_0, \Omega_{\text{m},0}, \Omega_{\sigma,0}, \Omega_{\text{l},0}$	3
4b	SSh2( $\Lambda_b = 0$ )	$H = H_0 \sqrt{\Omega_{\text{m},0}(1+z)^3 + \Omega_{\sigma,0} + 2\Omega_{\text{l},0} + 2\sqrt{\Omega_{\text{l},0}}\sqrt{\Omega_{\text{m},0}(1+z)^3 + \Omega_{\sigma,0} + \Omega_{\text{l},0}}}$	$H_0, \Omega_{\text{m},0}, \Omega_{\sigma,0}, \Omega_{\text{l},0}$	3

Table 1. The Hubble function and parameters for different models explaining acceleration in terms of dark energy.

where  $\mathcal{L}$  is the maximum likelihood and  $d$  is a number of the model parameters. The best model with a parameter set providing the preferred fit to the data is that minimizes the AIC. It is interesting that the AIC also arises from an approximate minimization of the Kulbak-Leibner information entropy [29].

The BIC introduced by Schwarz [15] is defined as

$$\text{BIC} = -2 \ln \mathcal{L} + d \ln N \quad (4)$$

where  $N$  is the number of data points used in the fit. The AIC tends to favour models with large number of parameters when compared to the BIC, so the latter provides a more useful approximation to the full statistical analysis in the case of no priors on the set of model parameters [28]. It makes this criterion especially suitable in context of cosmological applications.

It is pointed out that while the AIC is useful in obtaining upper limit to the number of parameters which should be incorporated to the model, the BIC is more conclusive. Of course only the relative value between BIC of different models has statistical significance. The difference of 2 is treated as a positive evidence (and 6 as a strong evidence) against the model with a larger value of the BIC [30,31]. If we do not find any positive evidence from information criteria the models are treated as identical and eventually additional parameters are treated as not significant. The using of the BIC seems to be especially suitable whenever the complexity of reference does not increase with the size of data set which is important in the context of the future SNAP observations. In a footnote Liddle [11] noted that in cosmology, a new parameter is usually a quantity set to zero in a simpler base model and if the likelihood function is a continuous function of its parameters it will increase as the parameter varies in either the positive or negative direction.

## 4 Distant supernovae as cosmological probes dark energy origin

In this section it will be demonstrated that using the Akaike and Bayesian information criteria one can answer which cosmological model is favoured? We consider only two types of models 1) a model with dark energy violating strong energy condition or 2) a model in which dark energy has brane origin. We use the “Gold” SNIa data set selected by Riess et al. [5]. This sample contains 157 SNIa with redshift up to  $z = 1.75$ , Recently Astier et al. [32] have compiled a new sample of 115 supernovae based on 71 high redshift type Ia supernovae discovered during the first year of the 5-year Supernovae Legacy Survey. Thanks to the multi-band, rolling search technique and careful calibration, this data set is arguably the best high- $z$  SNIa compiled data [32,33], unfortunately only with supernovae up to  $z = 1$ . It is the main reason that

we decide to use both SNIa samples in our analysis.

For the distant SNIa one can directly observe their apparent magnitude  $m$  and redshift  $z$ . Because the absolute magnitude  $\mathcal{M}$  of the supernovae is related to its absolute luminosity  $L$ , then the relation between the luminosity distance  $d_L$  and the observed magnitude  $m$  and the absolute magnitude  $M$  has the following form

$$m - M = 5 \log_{10} d_L + 25. \quad (5)$$

Instead of  $d_L$ , the dimensionless parameter  $D_L$

$$D_L = H_0 d_L \quad (6)$$

is usually used and then eq. (5) changes to

$$\mu \equiv m - M = 5 \log_{10} D_L + \mathcal{M} \quad (7)$$

where

$$\mathcal{M} = -5 \log_{10} H_0 + 25. \quad (8)$$

We know the absolute magnitude of SNIa from the light curve. The luminosity distance of a supernova can be obtain as the function of redshift

$$d_L(z) = (1+z) \frac{c}{H_0} \frac{1}{\sqrt{|\Omega_{k,0}|}} \mathcal{F} \left( H_0 \sqrt{|\Omega_{k,0}|} \int_0^z \frac{dz'}{H(z')} \right) \quad (9)$$

where  $\Omega_{k,0} = -\frac{k}{H_0^2}$  and

$$\begin{aligned} \mathcal{F}(x) &= \sinh(x) & \text{for } k < 0 \\ \mathcal{F}(x) &= x & \text{for } k = 0 \\ \mathcal{F}(x) &= \sin(x) & \text{for } k > 0. \end{aligned} \quad (10)$$

Finally, it is possible to probe dark energy which constitutes the main contribution to the matter content. It is assumed that supernovae measurements come with the uncorrelated Gaussian errors and in this case the likelihood function  $\mathcal{L}$  can be determined from the chi-square statistic  $\mathcal{L} \propto \exp(-\chi^2/2)$  where

$$\chi^2 = \sum_i \frac{(\mu_i^{\text{theor}} - \mu_i^{\text{obs}})^2}{\sigma_i^2}, \quad (11)$$

( $\sigma_i$  denotes the full statistical error of magnitude determination including error in  $z$  measurement) while the probability density function of cosmological parameters [3] is derived from Bayes' theorem. Therefore, we can perform the estimation of the model's parameters using the minimization procedure, based on the likelihood function.

The results of statistical calculation of considered dark energy models are presented in Table 2 and Table 3. In Table 2 we show results for models con-

sidered (flat cases) without any assumed extra priors. In Table 3 we presented analogous results for the case of the assumed prior  $\Omega_{m,0} = 0.3$  [34]. Since the value of matter density is not yet known precisely we also present results for  $\Omega_{m,0} = 0.2$  and  $\Omega_{m,0} = 0.4$ .

Using the best fit method based on the maximum of the likelihood function (minimum  $\chi^2$ ) we conclude that SSh models are a better fit than the  $\Lambda$ CDM model if we consider models without any assumed extra priors.

The above mentioned results are illustrated in Fig. 1. We present residual plots of the redshift-magnitude relations between the Einstein-de Sitter model (represented by zero line) the best-fitted SSh “Brane(1)” model (middle curve) and the flat  $\Lambda$ CDM model—the upper curve. Please note that best fitted “Brane (2)” model is inseparable from the  $\Lambda$ CDM model. These two models seems to be inseparable in the low and middle redshifts (see also [23]). One can observe that the systematic deviation between the  $\Lambda$ CDM model and the SSh “Brane(1)” model gets larger at higher redshifts ( $z > 0.9$ ). The SSh “Brane(1)” model predicts that high redshift supernovae should be brighter than predicted with the  $\Lambda$ CDM model.

We supplement our analysis of the SSh model with confidence levels intervals in the  $(\Omega_{m,0}, H_0)$  plane by marginalizing the probability density functions over remaining parameters assuming uniform priors. (Fig. 2). We obtain that the SSh “Brane (1)” model prefers universes with high density of  $\Omega_{m,0} \simeq 0.75$ , while the SSh “Brane (2)” model prefers universes with low density of  $\Omega_{m,0} \simeq 0.20$ . It is a situation contrary to the  $\Lambda$ CDM where  $\Omega_{m,0} \simeq 0.3$  is preferred [3,4,5].

The results of the AIC and BIC in the context of considered models (Table 1) are collected in Tables 2 and 3. One can observe that the BIC as well as AIC values assumes lower values for the  $\Lambda$ CDM model when we do not assumed any prior for  $\Omega_{m,0}$ .

For deeper statistical analysis dependence of  $\chi^2$  value, the AIC and BIC on  $\Omega_{m,0}$  is presented in Fig. 3–5. These figures was obtained fixing  $\Omega_{m,0}$  and than calculating  $\chi^2$  value, the AIC and BIC quantities for all points separately. Please note that if we fix  $\Omega_{m,0}$ , then the numbers of the free models parameters is less by one.

In Fig. 3 we present  $\chi^2$  values with respect to fixed  $\Omega_{m,0}$  for the Riess et al. and Astier et al. samples. We find no difference between  $\chi^2$  values of the SSh “Brane (1)” model, SSH1( $\Lambda_b = 0$ ) model and the  $\Lambda$ CDM model when  $\Omega_{m,0} \leq 0.31$  ( $\Omega_{m,0} \leq 0.26$  for Astier et al. sample) while this difference diverges for greater  $\Omega_{m,0}$ . The opposite situation is for the SSh “Brane (2)” model where it differs from the  $\Lambda$ CDM and SSH2( $\Lambda_b = 0$ ) in  $\chi^2$  for  $\Omega_{m,0} \leq 0.3$  and it has no differences when  $\Omega_{m,0} > 0.3$  ( $\Omega_{m,0} > 0.26$  for the Astier et al. sample).



From this analysis we of course obtain the conclusion made by Alam and Sahni [19] that the SSh model fits the SNIa data better than the  $\Lambda$ CDM model (with exception  $\Omega_{m,0} \simeq 0.3$  for the Gold sample and  $\Omega_{m,0} \simeq 0.26$  for the Astier et al. sample. Please note that only for the SSh “Brane (1)” model analysed with the Gold sample we obtain that the value of  $\chi^2$  is significantly lower than for the  $\Lambda$ CDM model.

Results obtained with both Riess et al. and Astier et al. samples are similar. The main difference lies in best fitted values for  $\Omega_{m,0}$ . Also results obtained from the Riess et al. sample show the advantage of the SSh “Brane (1)” model over SSh1( $\Lambda_b = 0$ ) model for  $\Omega_{m,0} > 0.31$  while from the Astier et al. sample do not show such preferences.

When we analyse “Brane (1)” model with the Riess et al. SNIa sample, in Fig. 3–5 one can observe the characteristic “knee” for the value  $\Omega_{m,0} \simeq 0.3$ . This effect comes from the fact that statistical analysis of the SSh “Brane (1)” model for  $\Omega_{m,0} < 0.31$  gives close to zero value of  $\Omega_{l,0}$  as a best fit. It means that in this interval of  $\Omega_{m,0}$  the influence of additional parameters is small or negligible. The analogous effect takes place for the “Brane (2)” solution in the interval  $\Omega_{m,0} > 0.3$ . Please note that only for the SSh “Brane (1)” model analysed with the Gold sample we obtain that the value of  $\chi^2$  is significantly lower than for the  $\Lambda$ CDM model. Please note that Alam and Sahni (see Fig. 1 in [27]) did not find “knee” behaviour for “Brane (1)” model. The reason is that in our analysis the parameter  $\Omega_{\Lambda_b}$  is free while in Alam and Sahni’s paper this parameter is estimated and then fixed.

The most recent WMAP data [35] seem to prefer the matter density about  $\Omega_{m,0} \simeq 0.24$  which is lower than canonical  $\Omega_{m,0} = 0.3$ . In this case we obtain that the lowest value of  $\chi^2$  we obtain for “Brane (2)” model. Moreover when we analyse the Riess et al. sample the value of  $\chi^2$  for both  $\Lambda$ CDM and DDG models are equal.

This preference of the SSh model over the  $\Lambda$ CDM model is not confirmed by information criteria. With both the AIC and BIC criteria we obtain that the model which minimizes both the AIC and BIC is the  $\Lambda$ CDM model. There is also a significant difference between predictions of these models. The  $\Lambda$ CDM model prefers a universe with  $\Omega_{m,0}$  close to 0.3, the SSh “Brane (1)” model favours a high density universe, while the DDG model and the SSh “Brane (2)” model favour a low density universe. In Fig. 4 and Fig. 5 we present values of the AIC and BIC for the considered models. If  $\Omega_{m,0} \in (0.15, 0.24)$  then the information criteria favour the DDG model, while for  $\Omega_{m,0} < 0.15$  ( $\Omega_{m,0} < 0.11$  from BIC) the SSh “Brane (2)” model is favoured. For  $\Omega_{m,0} \in (0.24, 0.37)$ , the  $\Lambda$ CDM is favoured while for  $\Omega_{m,0} > 0.37$  ( $\Omega_{m,0} > 0.42$  when the BIC is considered) the SSh “Brane (1)” model is preferred over  $\Lambda$ CDM model. However, let us note that the value of  $\Omega_{m,0} \geq 0.4$  seems to be too high

Table 2

The values of the  $\chi^2$ , the AIC and BIC for the models from Table 1.

case	Riess et al.					Astier et al.				
	$\chi^2$	AIC	BIC	$\Omega_{m,0}$	$\chi^2/\text{dof}$	$\chi^2$	AIC	BIC	$\Omega_{m,0}$	$\chi^2/\text{dof}$
1	175.9	179.9	186.0	0.31	1.135	107.8	111.8	117.3	0.26	0.954
2	176.9	180.9	187.0	0.20	1.141	108.0	112.0	117.5	0.17	0.956
3a	172.3	180.3	192.5	0.53	1.126	107.7	115.7	126.7	0.51	0.970
3b	175.8	183.8	196.0	0.30	1.149	107.8	115.7	126.8	0.26	0.971
4a	174.8	180.8	189.9	1.00	1.135	107.7	113.7	121.9	1.00	0.962
4b	175.9	181.9	191.0	0.310	1.142	107.8	113.8	122.0	0.26	0.962

in comparison with the present extragalactic data [34]. When we analysed Astier et al. sample these values a little change because of difference in best fitted values for  $\Omega_{m,0}$  especially for  $\Lambda$ CDM model. For example the  $\Lambda$ CDM model is favoured for  $\Omega_{m,0} \in (0.21, 0.31)$  when the AIC is considered and for  $\Omega_{m,0} \in (0.21, 0.35)$  when we consider BIC.

The BIC (and also the AIC in the case of Astier et al. sample) show preferences the SSh1( $\Lambda_b = 0$ ) model over the SSh “Brane (1)”. Both BIC and AIC show preferences the SSh2( $\Lambda_b = 0$ ) model over the SSh “Brane (2)”. However the information criteria still favour the  $\Lambda$ CDM over SSh1( $\Lambda_b = 0$ ) model if  $\Omega_{m,0} < 0.36$  in the case of the AIC ( $\Omega_{m,0} < 0.31$  for Astier et al. sample) and  $\Omega_{m,0} < 0.4$  ( $\Omega_{m,0} < 0.35$  for Astier sample) in the case of the BIC.

With no prior on the value of matter content  $\Omega_{m,0}$  the information criteria favour the  $\Lambda$ CDM model rather than the FRW brane models with extra dimensions. The same result in favour of the  $\Lambda$ CDM model is obtained if  $\Omega_{m,0}$  is not significantly different from the canonical value  $\Omega_{m,0} = 0.3$ . However, taking the other value of  $\Omega_{m,0}$  from independent observations (e.g. recent WMAP data —  $\Omega_{m,0} \simeq 0.24$  [35]) appears to equally favour the  $\Lambda$ CDM and DDG models (also SSh2( $\Lambda_b = 0$ ) if we consider AIC only) with the Gold Riess SNIa sample. Please also note that if allowed non flat models, non flat  $\Lambda$ CDM and DDG models are again equally favoured by information criteria [22].

Recently Eisenstein et al. have analysed the baryon oscillation peaks (BOP) detected in the SDSS Luminosity Red Galaxies Survey [36]. They found that

Table 3

The values of the  $\chi^2$ , the AIC and BIC for the models from Table 1 with the prior  $\Omega_{m,0} = 0.2, 0.3, 0.4$  obtained with the Gold from Riess et al. (denoted as  $G$ ) and Astier et al. (denoted as  $A$ ) samples.

	$\Omega_{m,0} = 0.2$				$\Omega_{m,0} = 0.3$				$\Omega_{m,0} = 0.4$			
case	$\chi^2$	AIC	BIC	$\chi^2/\text{dof}$	$\chi^2$	AIC	BIC	$\chi^2/\text{dof}$	$\chi^2$	AIC	BIC	$\chi^2/\text{dof}$
G1	185.7	187.7	190.8	1.190	175.9	177.9	181.0	1.128	180.8	182.8	185.9	1.159
G2	176.9	178.9	182.0	1.134	183.9	185.9	189.0	1.179	200.0	202.0	205.1	1.282
G3a	185.7	191.7	200.8	1.205	175.9	181.9	191.0	1.142	173.4	179.4	188.5	1.126
G3b	176.5	182.5	191.6	1.146	175.8	181.8	190.9	1.142	180.8	186.8	195.9	1.174
G4a	185.7	189.7	195.8	1.198	175.9	179.9	186.0	1.135	175.4	179.4	185.5	1.131
G4b	176.9	190.9	197.0	1.141	175.9	179.9	186.0	1.135	180.8	184.8	190.9	1.166
A1	110.9	112.9	115.6	0.973	108.8	110.8	113.5	0.954	119.4	121.4	124.1	1.047
A2	109.4	111.4	114.1	0.960	130.7	132.9	135.4	1.147	181.1	183.1	185.8	1.589
A3a	110.9	116.9	125.1	0.990	107.8	113.8	122.0	0.962	107.8	113.8	122.0	0.962
A3b	107.8	113.8	122.0	0.963	108.8	114.8	123.0	0.971	119.4	125.4	133.7	1.066
A4a	110.9	114.9	120.4	0.981	107.8	111.8	117.2	0.954	107.8	111.8	117.3	0.954
A4b	107.8	111.8	117.3	0.954	108.8	112.8	118.3	0.963	119.4	123.4	118.9	1.057

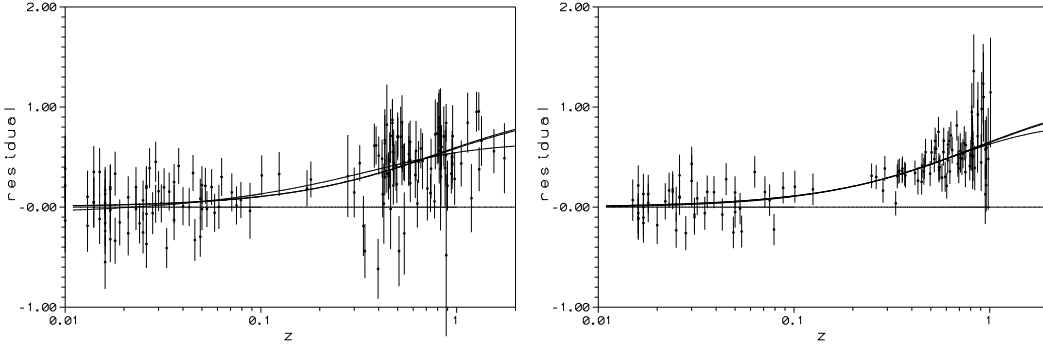


Fig. 1. Residual plots of the redshift-magnitude relations between the Einstein-de Sitter model (represented by the zero line) the best-fitted SSh “Brane (1)” model (middle curve) and the flat  $\Lambda$ CDM model—the upper curve. Best fitted SSh “Brane 2” model is inseparable from the  $\Lambda$ CDM model. Left panel was obtain with the Gold sample while the right panel was obtained with Astier et al.’s sample.

value of  $A$

$$A \equiv \frac{\sqrt{\Omega_{m,0}}}{E(z_1)^{\frac{1}{3}}} \left( \frac{1}{z_1 \sqrt{|\Omega_{k,0}|}} \mathcal{F} \left( \sqrt{|\Omega_{k,0}|} \int_0^{z_1} \frac{dz}{E(z)} \right) \right)^{\frac{2}{3}} \quad (12)$$

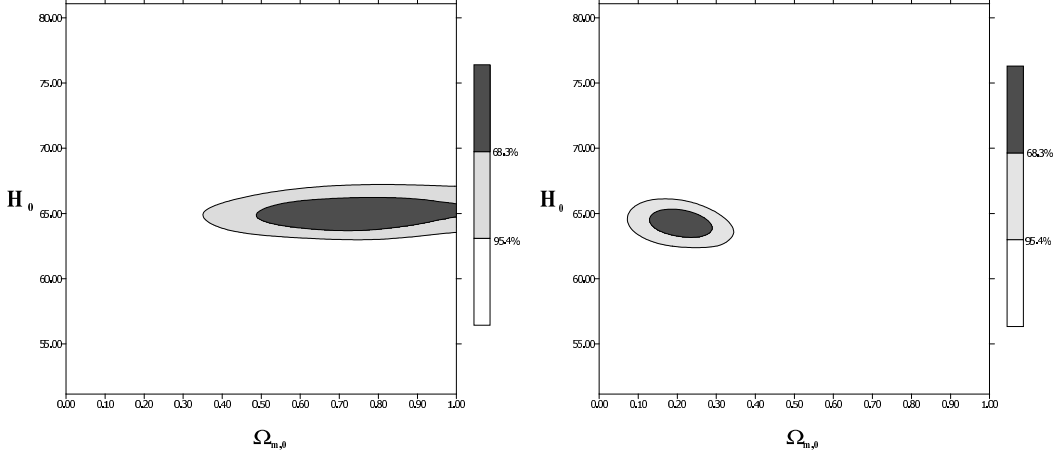


Fig. 2. Confidence levels intervals for the SSh “Brane (1)” (left panel) and “Brane (2)” (right panel) models on the  $(\Omega_{m,0}, H_0)$  plane. They are obtained with the Gold sample by marginalizing the probability density functions over remaining parameters (assuming uniform priors).

where  $E(z) \equiv H(z)/H_0$  and  $z_1 = 0.35$  is equal  $A = 0.469 \pm 0.017$ . The quoted uncertainty corresponds to one standard deviation, where a Gaussian probability distribution has been assumed. This constraints could also be used for fitting cosmological parameters [32,33,27,37,38]. The cosmological parameters are derived from Bayes’ theorem in the standard way [3]. Estimated value of  $\Omega_{m,0}$  as well as confidence levels intervals for  $\Omega_{m,0}$  we obtain by marginalizing the probability density functions over remaining parameters, assuming uniform priors. The estimated value of the model parameter  $\Omega_{m,0}$  as well as 95% confidence interval of  $\Omega_{m,0}$  from the BOP is presented in Table 4. Please note that for both SSh “Brane (1)” and SSh “Brane (2)” models, the 95% confidence level regions (obtained with the BOP analysis) and the regions of  $\Omega_{m,0}$  at which these models are favoured by information criteria, are disjoint. Therefore from the joint analysis of the BOP and information criteria we obtain that the  $\Lambda$ CDM model is still favoured over both the SSh “Brane (1)” and “Brane (2)” models. Our analysis also confirm previous conclusion in Ref. [33] that the flat DDG model can be virtually ruled out by statistical analysis. Our analysis allowed only that the SSh1( $\Lambda_b = 0$ ) model could be preferred over  $\Lambda$ CDM model if  $\Omega_{m,0} \simeq 0.4$ .

## 5 Conclusion

The main goal of this letter is to decide which class of models with dark energy are distinguished by statistical analysis of SNIa data. For this aim the Akaike and Bayesian information criteria are adopted. Two categories of the models were considered, one with dark energy in the form of fluid violating strong

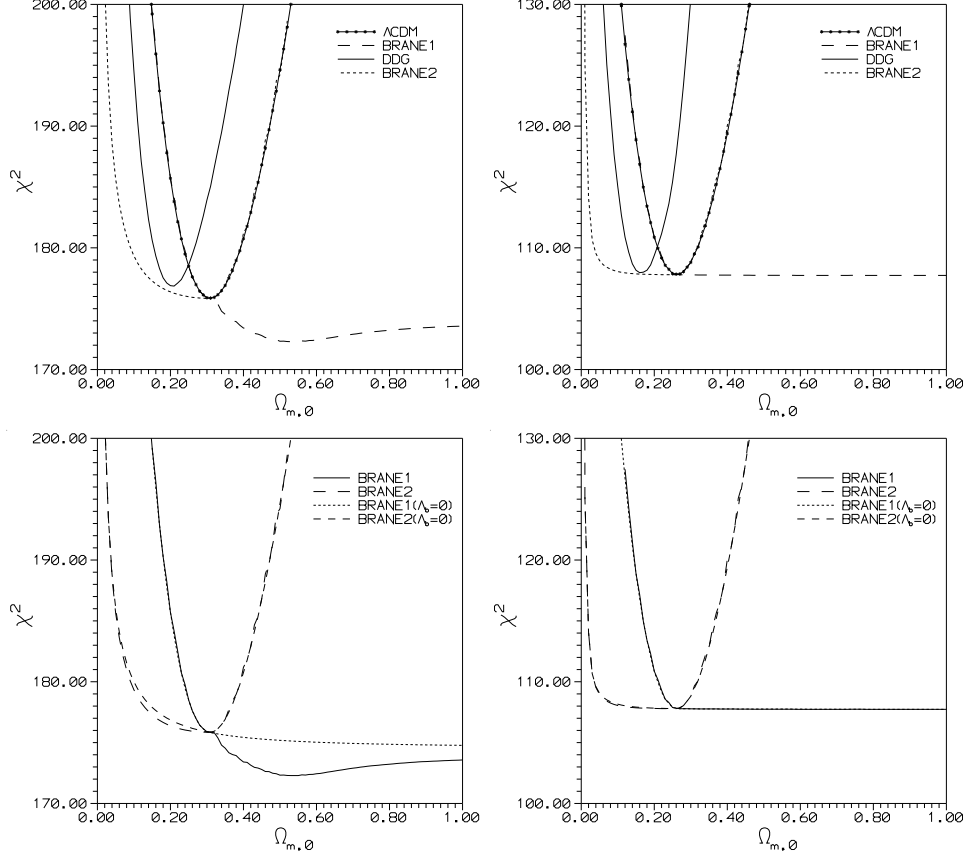


Fig. 3. The values of  $\chi^2$  with respect to the value of  $\Omega_{m,0}$  for considered models, marginalized over remaining model parameters. The left panel was obtained with the Gold sample while right panel was obtained with Astier et al.'s sample. We find no difference between  $\chi^2$  values of the SSh “Brane (1)” model and the  $\Lambda$ CDM model when  $\Omega_{m,0} \leq 0.31$  ( $\Omega_{m,0} \leq 0.26$  for the Astier et al. sample). The opposite case is for the SSh “Brane (2)” model where it does not differ from the  $\Lambda$ CDM when  $\Omega_{m,0} > 0.3$  ( $\Omega_{m,0} > 0.26$  for Astier et al. sample). Please also not that for the Astier et al. sample SSh “Brane” models are inseperable form SSh( $\Lambda_b = 0$ ) models.

Table 4

The constraints for  $\Omega_{m,0}$  from the baryon oscillation peak test for the models from Table 1. We present best fitted value as well as 95% confidence interval.

case	$\Omega_{m,0}$	95% level
1	0.273	(0.228,0.326)
2	0.305	(0.252,0.363)
3a	0.23	(0.16,0.28)
3b	0.32	(0.25,0.49)
4a	0.298	(0.251,0.358)
4b	0.367	(0.297,0.498)

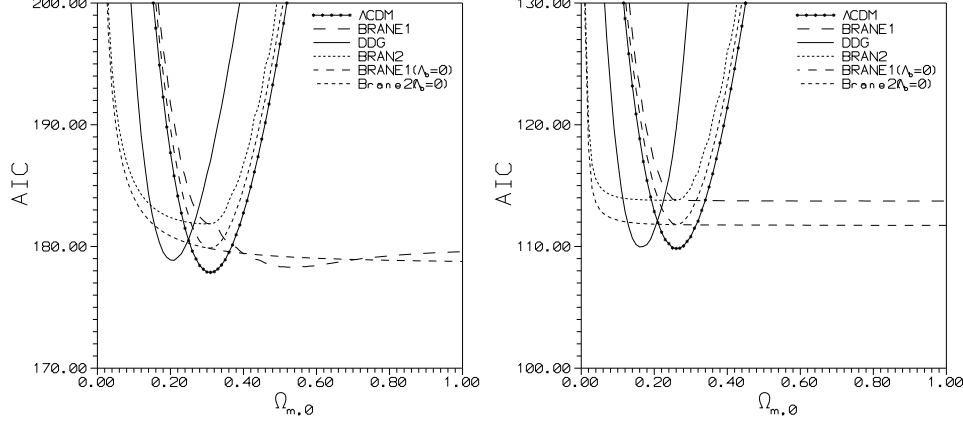


Fig. 4. The values of the AIC for the Gold sample from Riess et al. (left panel) and Astier et al.'s sample (right panel) with respect to the value of  $\Omega_{m,0}$  for the considered models.

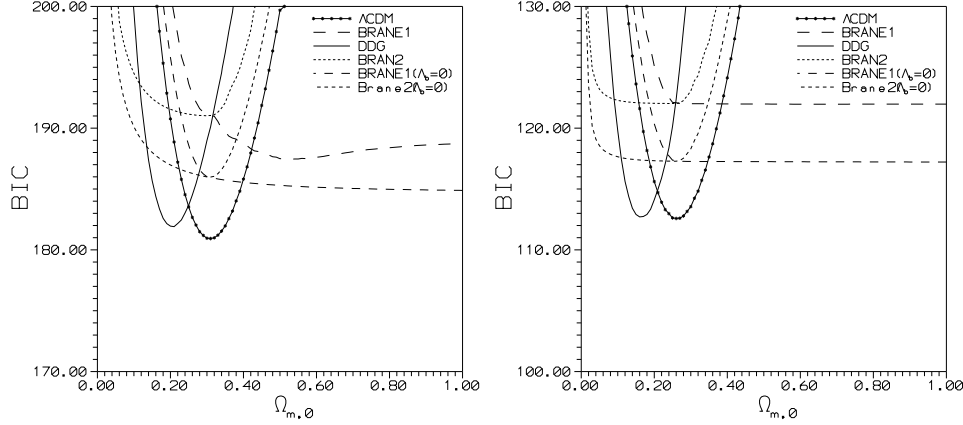


Fig. 5. The values of the BIC for the Gold sample from Riess et al. (left panel) and Astier et al.'s sample (right panel) with respect to the value of  $\Omega_{m,0}$  for the considered models.

energy condition and second in which dark energy is the present manifestation of embedding a brane (our universe) in a larger, higher dimensional bulk space. One concludes that both the AIC and BIC weigh in favour of the models of the first category (with the  $\Lambda$ CDM as their representative case) over the FRW brane model with extra dimensions. Assuming the prior  $\Omega_{m,0} = 0.3$  both the AIC and BIC weigh in favour of the  $\Lambda$ CDM model. However please note the most recent WMAP data [35] seem to favour a lower value of matter density  $\Omega_{m,0} \simeq 0.24$ . For such value of  $\Omega_{m,0}$  at least during the analysis with the Gold Riess et al. SNIa sample both the AIC and BIC equally favour the  $\Lambda$ CDM and DDG models. Please also note that if allowed non flat models, the non flat  $\Lambda$ CDM and DDG models are again equally favoured by the information criteria [22].

The further conclusions are the following.

- If we consider models in which all model parameters are fitted then the  $\Lambda$ CDM model is preferred. DDG model give a greater value of  $\chi^2$  over the  $\Lambda$ CDM model. Both the SSh “Brane (1)”, “Brane (2)” as well as SSh( $\Lambda_b = 0$ ) models, under the similar quality of the fit as for the  $\Lambda$ CDM model, contains more parameters than the  $\Lambda$ CDM model.
- When we consider the prior on  $\Omega_{m,0}$  then for  $\Omega_{m,0} < 0.24$  the DDG model is favoured by the information criteria over the  $\Lambda$ CDM model while for the “normal” density universe with  $\Omega_{m,0} \simeq 0.3$  the  $\Lambda$ CDM model is favoured. Only for the high density universe ( $\Omega_{m,0} > 0.38$  from the AIC and  $\Omega_{m,0} > 0.42$  from the BIC) is the SSh “Brane (1)” model preferred over the  $\Lambda$ CDM model. The SSh “Brane (2)” model is preferred for the low density universe ( $\Omega_{m,0} < 0.15$  from the AIC and  $\Omega_{m,0} < 0.11$  from the BIC). When we analyse Astier et al. sample these values a little change. The  $\Lambda$ CDM model is favoured for  $\Omega_{m,0} \in (0.21, 0.31)$  when we consider AIC and for  $\Omega_{m,0} \in (0.21, 0.35)$  when we consider BIC.
- The BIC information criterion favours the SSh1( $\Lambda_b = 0$ ) model over the  $\Lambda$ CDM for  $\Omega_{m,0} > 0.4$  ( $\Omega_{m,0} > 0.36$  for the Astier et al. sample). The AIC favour the SSh1( $\Lambda_b = 0$ ) model even for  $\Omega_{m,0} > 0.36$  ( $\Omega_{m,0} > 0.31$  for the Astier et al. sample).
- If we compare SSh “Brane ” models with SSh( $\Lambda_b = 0$ ) models using the BIC than we obtain that the SSh( $\Lambda_b = 0$ ) model is preferred over SSh “Brane ” models.
- If we consider the 95% confidence level regions obtained with the BOP analysis and the region of  $\Omega_{m,0}$  where model “Brane (1)” and “Brane (2)” are favoured by the information criteria we find that they are disjoint. It means that the  $\Lambda$ CDM model is still favoured over both SSh “Brane (1)” and “Brane (2)” models from joint analysis.
- We find that the  $\Lambda$ GDP model is preferred over the  $\Lambda$ CDM model by the joint analysis but only if  $\Omega_{m,0}$  is close to 0.4.

We find no difference between  $\chi^2$  values of the SSh “Brane (1)” model and the  $\Lambda$ CDM model when  $\Omega_{m,0} \leq 0.31$  ( $\Omega_{m,0} \leq 0.26$  for the Astier et al. sample). The opposite situation is for the SSh “Brane (2)” model where and it has no differences when  $\Omega_{m,0} > 0.3$  ( $\Omega_{m,0} > 0.26$  for the Astier et al. sample). In these intervals the SSh “Brane (1)” and SSh “Brane (2)” models are, therefore, indistinguishable from the concordance  $\Lambda$ CDM model. And we have some kind of dynamical equivalence between two pairs of models in the above found intervals. This lack of differentiation in terms of  $\chi^2$  can be overcome by the Akaike and Bayesian information criteria (see Fig. 4 and Fig. 5).

In observational cosmology we are encountered with the so-called degeneracy problem which consists in the existence of many theoretical models with dramatically different cosmological scenarios (big bang versus bounce, big rip versus de Sitter phase, etc.) but in a good agreement with the current observations. A nice way of overcoming this problem seems to be adopting the

information criterion approach. Since it provides a very simple and objective criterion for the inclusion of additional parameters in the cosmological model, it could be used for model selection instead of the best-fit method. In context of dark energy the information criteria give us information if the present observational data suggest taking into account new degrees of freedom. Of course in any case introducing a new term can be suggested in the other way – for example from theoretical prediction.

Our general conclusion is that the high precision detection of distant type Ia supernovae could justify an inclusion of new parameters related with embedding our Universe in bulk space. Our results were obtained from SNIA data set and baryon oscillation peak. However, to make the final decision which model describes our Universe it is necessary to obtain the precise value of  $\Omega_{m,0}$  from independent observations. Other future investigations such as gravitational lensing, WMAP, X-ray gas mass fraction measurements are required for the final resolution of the problem.

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